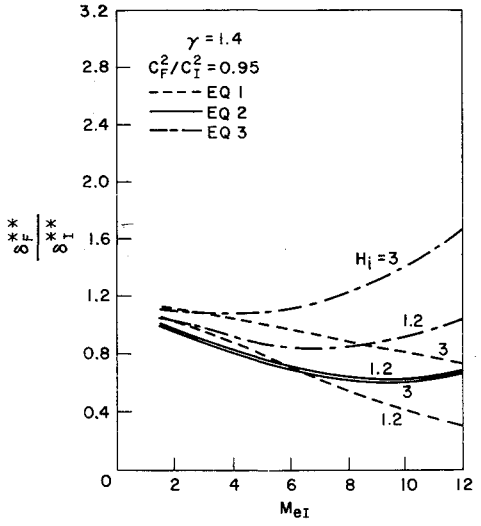
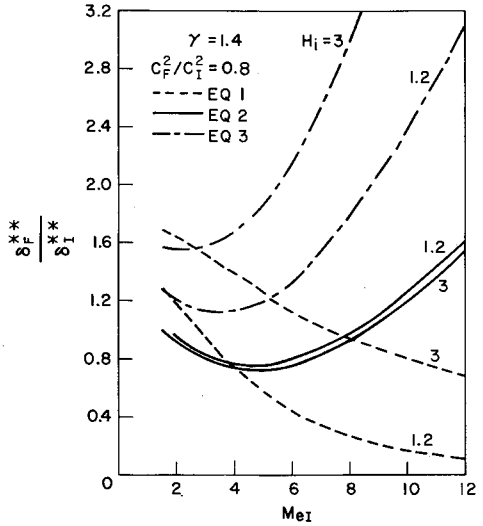


a) $C_F^2/C_I^2 = 0.95$



b) $C_F^2/C_I^2 = 0.80$

Fig. 2 Effect of an abrupt compression on boundary-layer momentum thickness.

Equations (1–3) are plotted in Figs. 2a and 2b. These plots were made for two compression ratios ($C_F^2/C_I^2 = 0.95$ and 0.80) and two values of the incompressible shape factor ($H_i = 1.2$ and $H_i = 3$).‡ θ_w and θ_2 are calculated for a given compression ratio and initial Mach number by using the standard oblique shock relationships.

As shown in Fig. 2, Eq. (1) predicts a monotonic decrease in $\delta_F^{**}/\delta_I^{**}$ as M_{eI} increases for a fixed compression ratio C_F^2/C_I^2. Equations (2) and (3) both show a different qualitative trend, i.e., $\delta_F^{**}/\delta_I^{**}$ decreases at first and then increases as M_{eI} increases for a fixed compression ratio.

All three equations show qualitative agreement with White's experiments[1] at a compression ratio of approximately 0.87, where $\delta_F^{**}/\delta_I^{**}$ was between 1.11 and 0.99 for M_{eI} between 2.5 and 2.9. The fact that Eq. (2) is relatively insensitive to the choice of H_i (an unknown) makes it more attractive than Eqs. (1) and (3) which are quite sensitive to the chosen H_i. However, from Fig. 2 it is obvious that further experiments are needed to select the superior approximation.

‡ The actual value of H_i at separation is unknown for supersonic flow. However, these limits should cover the range of possible values since Schlicting[6] gives the following for incompressible flow; $1.8 \leq H_i \leq 2.4$.

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Time Dilatation in Relativity

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THE article on time dilatation by v. Krzywoblocki[1] in the AIAA Journal, in which the author claims to resolve the clock paradox, calls for the following comments. His arguments are based mainly on the following three points: 1) The possibility of two processes, the Clausius process and the Lorentz process, the independence of the former frame from relativistic transformations. 2) The internal mechanism of a clock belongs to a Clausius family; i.e., the light signals do not affect the action or the rate of action of the mechanism of the clock. 3) Space trip clocks may be constructed with a different scale, i.e., a unit of time γ^{-1} times that of the earth clock.

Assumption 1 is ruled out in relativity theory, as no absolute frames or absolute processes independent of the frames of references, are supposed to exist. If assumption 2 is valid, that is, if the mechanism is not affected, it is not correct to introduce 3. Therefore, 2 and 3 appear to be contradictory.

We may now proceed with the following comments:

a) That the rate of clock should be adjusted to coincide with a fixed clock is valid in pre-Einstein theory only, since absolute time is itself ruled out.

b) No reference appears anywhere in the discussion of time dilatation about the effects of velocity or acceleration on the movement or mechanism of a clock.

c) For experiments involving high velocities, only atomic clocks, such as the vibration of the Caesium atom, are used. The possibility of adjustment of rate does not arise.

d) If the clock in S' is retarded relative to S, then the clock in S is retarded relative to S'; the adjustment of one does not solve the riddle.

e) If S moves with velocity v relative to S_1, and with velocity v_2 relative to S_2, and if S, with the adjusted clock, leaves S_1 and after some time passes through S_2, then the synchronization of S_1 and S_2 will be complicated as it is not working at a normal rate.

f) Finally, the question of adjustment assumes prepared journeys, as in the case of space travel. But in physics we do not deal with prepared journeys, but will have to explain the relations between the journeys, not necessarily planned by us.

It is better that the paradox is left as it is, as all the experimental evidence has very well supported Einstein's special theory of relativity. The solution of the paradox does not lie

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in inventing situations to explain its absence. A revolutionary change in the fundamental concepts of physics, or a suitable modification of relativity theory, will solve the riddle in the future.

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Reply by Author to V. S Ananthachar

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A READER approaching the field of special relativity must keep in mind particularly two items: 1) no scientist ever stated or attempted to state that all of the processes in nature must obey the postulates of the Einstein special theory of relativity; 2) there exists more than one special theory of relativity. For more details concerning 1, the reader is referred to works by Rothman,[1] Schlegel,[2] and others. Concerning 2, a very profound theory of relativity of Whitehead[3] is seldom mentioned, and was applied to only a few cases. It may be of interest to acquaint oneself with the opinion of Whitehead on the subject of the theories of relativity. Whitehead wrote, "...Einstein, in my opinion, leaves the whole antecedent theory of measurement in confusion, when it is confronted with the actual conditions of our perceptual knowledge." In another place he says, "The possibility of other such laws, expressed in sets of differential equations other than Einstein's, arises from the fact that in my theory there is a relevant fact of nature which is absent on Einstein's theory." And, "The course of my argument has led me generally to couple my allusions to Einstein with some criticism.... But the worst homage we can pay to a genius is to accept uncritically formulations of truths which we owe to it (i.e., a method due to Einstein)." Recently, Schlegel[2] proposed his formulation of the theory of relativity, of Clausius and Lorentz (relativistic) processes. It is the expression of a deep intuition of a theoretical physicist who feels that the nature and the universe are not ruled autocratically by the geometrically oriented laws discovered by science in the past and that there are other laws which we do not know, as yet. It is known that some leading scientists in the field are a little disappointed with the purely geometrically oriented approach to model, and to describe, the laws of nature.† In this, Schlegel differs from both Whitehead and Einstein, and I do not hesitate to call his approach "Schlegel's formulation of the theory of relativity." Schlegel's formulation was nicely accepted. In the title of my note, published in 1964, I do not use the words "relativity, Einstein, etc." In the first two sections of it, I have presented a few general remarks. In the third section, entitled "Scale Variation in the Time-Dilatation Dilemma," the first sentence is "We accept the validity of the Schlegel hypothesis." In this section, there does not appear the name of Einstein even once. The contents of the section consist of the discussion of a particular example that illustrates Schlegel's idea (but not mine). I have shown in this example that by using Schlegel's formulation, it is possible to synchronize two clocks, one on the earth, another on an earth bound satellite, without the appearance of the so-called clock-paradox (should be rather the clock-ambiguity, as Schlegel has demonstrated). In this section there are no remarks about cosmological aspects whatsoever; there are presented no new ideas, no new fundamentals. Referring to Ananthachar's remarks, I must state that I fail to see what he wants to say with reference to my note. He states that, "Assumption 1 is ruled out in relativity theory,..." It is not said in which relativity theory. Certainly not in the Schlegel theory, where this is one of the main assumptions. Hence this remark cannot be referred to my note. Ananthachar's Comment "A" refers to pre-Einstein theory. Comment "B" is very thoroughly explained in Schlegel's book, to which the reader is referred. Comment "C" refers to the kind of atomic clock; this point is not even mentioned by me. Comment "D" is discussed very thoroughly in Schlegel's book. Comment "E" refers to the situation when S_1, after some time, passes through S_2. Practically, this means that the spaceship will return to the earth and obviously the matter is closed. In Comment "F" Ananthacar states that my case assumes prepared journeys, as in the case of space travel. But I exactly discuss only the case of space travel. I do not discuss any case of journeys "... not necessarily planned by us..." (quoting Ananthachar). In the last paragraph, Ananthachar expresses his point of view; it is better that the paradox is left as it is. I absolutely never had and now have no objections to leaving the paradox in the Einstein theory of relativity as it is.

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† The recent theory of R. Dicke from Princeton demonstrates very clearly that the geometry in the theory of relativity is completely unnecessary. A scalar and a tensor fields are completely sufficient.

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Comments on "Approximate Analytical Solution for Satellite Orbits Subjected to Small Thrust or Drag"

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IN a current paper by Brofman[1] the author has investigated the problems of satellites subjected to small thrust and drag. Both of these problems were previously studied by the writer.[2–4]

For the small tangential thrust problem, the expression of θ presented in Ref. 2 can be greatly improved by combining Eqs. (3, 6, 23, and 39) in Ref. 2 and integrating the resulting expression as†

$$\theta = a\tau - a^2A\{3[X(\ln X - 1) + 1] + 3a[X(\ln X - 1)^2 + (X - 2)] + a^2[X(\ln Y - 1)^3 + 3X(\ln X - 1) - (2X - 6)]\} \quad (1)$$

where the initial condition $\tau = 0$ and $\theta = 0$ has been employed, and

$$a \equiv \omega\bar{c}/g_0 \qquad \text{and} \qquad X \equiv 1 - (\tau/A)$$

Equation (1) has been shown elsewhere[5] and its accuracy against numerical integration was depicted in Ref. 5. With θ shown in Eq. (1) instead of Eq. (37) in Ref. 2, it is thought

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† See Nomenclature list in Ref. 2.